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Bounded Solutions of the Dirac Equation with a PT-symmetric Kink-Like Vector Potential in Two-Dimensional Space-Time

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Abstract The $(1+1)$-dimensional Dirac equation with a PT-symmetric kink-like vector potential is investigated. By using the basic concepts of the supersymmetric WKB formalism and the function analysis method, we solve exactly the Dirac equation and obtain the bound-state energy levels and two-component spinor components. The PT-symmetric kink-like potential is not Hermitian and absent of bound states in the context of non-relativistic Schrödinger equation, but it possesses two sets of real discrete relativistic energy spectra in the context of the Dirac theory. When the PT symmetry is spontaneously broken, two sets of real energy spectra come into complex conjugate.

Keywords Dirac equation · PT symmetry · Kink-like potential

1 Introduction

In a pioneering letter, Bender and Boettcher [1] observed that a non-Hermitian quantum system that exhibits PT symmetry can has a real energy spectrum when the PT symmetry is not spontaneously broken. Under the transformation of $x \to -x$ (or $x \to \xi - x$) and $i \to -i$, if a potential satisfies the relation $V(-x) = V^*(x)$ or $V(\xi - x) = V^*(x)$, the potential $V(x)$ is said to hold PT symmetry, where P represents parity reflection and T represents time reversal. In recent years, many PT-symmetric models have been examined in detail within the framework of non-relativistic quantum mechanics [2–24]. Non-Hermitian but PT-symmetric models have many applications in different research fields, such as nuclear physics [25, 26], quantum field theories [27–29], electromagnetic wave traveling in a planar slab waveguide [30].

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In recent years, some authors [31–40] investigated the PT-symmetric potential models in the context of relativistic quantum mechanics. With the help of the Nikiforov-Uvarov method, Simsek and Egrifes [31] studied the bound states of the $(1+1)$-dimensional Klein-Gordon equation with the PT-symmetric generalized vector Hulthén potential. Egrifes and Sever [32] investigated the bound states of the Dirac equation with the PT-symmetric generalized vector Hulthén potential in $1+1$ dimensions. In [33], Egrifes and Sever [33] investigated the $(1+1)$-dimensional Klein-Gordon with the scalar potential coupling scheme for the PT-symmetric generalized Hulthén potential. For the PT-symmetric versions of the Rosen-Morse well, Eckart potential and Scarf II potential, the s-wave bound states of the Klein-Gordon equation with equal scalar and vector potentials have been investigated by using the basic concepts of the supersymmetric quantum mechanics formalism and function analysis method [34, 35]. Sinha and Roy [36] investigated the bound state solutions of the $(1+1)$-dimensional Dirac equation with a non-Hermitian pseudoscalar potential, which reverses its sign under PT transformation. In [37], the authors presented a new procedure to construct the one-dimensional non-Hermitian imaginary potential in the setting of the position-dependent effective mass Dirac equation with the Lorentz vector potential in $1+1$ dimensions. Mustafa and Mazharimousavi [38] gave a valuable comment on the work [37]. Following up of the work [37], a new method has been proposed to construct the exactly solvable PT-symmetric potentials within the framework of the position-dependent effective mass $(1+1)$-dimensional Dirac equation with the vector potential coupling scheme [39]. By using the method proposed in [37], the relativistic problem of neutral fermions subject to PT-symmetric trigonometric potential ($\sim i\alpha \tan \alpha x$) in $1+1$ dimensions has been investigated [40].

Recently, de Castro and Hott [41] investigated the relativistic problem of neutral fermions subject to a pseudoscalar kink-like potential ($\sim \tanh \alpha x$). This parity-conserving pseudoscalar potential is of interest in quantum field theory where a classically stable and a finite localized energy solution of the motion equation can be in topologically stable sectors. Models of these kinds, known as kink models are obtained in quantum field theory as the continuum limit of linear polymer models [42, 43]. For this kink-like potential, there exists no bound state in a non-relativistic quantum theory because it gives rise to a ubiquitous repulsive potential. However, bound states of the this kink-like potential exist in $(1+1)$-dimensional Dirac equation for a fermion coupled to a pseudoscalar potential [41]. In the present work, we investigated the relativistic problem of neutral fermions subject to the PT-symmetric version of a kink-like potential. The exact solutions of the $(1+1)$-dimensional Dirac equation with the PT-symmetric kink-like vector potential are obtained in terms of the basic concepts of the supersymmetric WKB formalism and the function analysis method.

2 $(1+1)$-Dimensional Dirac Equation with a PT-symmetric Vector Potential

The $(1+1)$-dimensional time-independent Dirac equation for a fermion of rest mass M coupled to a vector potential $V(x)$ reads

$$(\alpha p + \beta m + V)\Psi(x) = E\Psi(x), \tag{1}$$

where E is the energy of the fermion, p is the momentum operator, and α, β are 2×2 matrices satisfying $\alpha^2 = \beta^2 = 1$, $\{\alpha, \beta\} = 0$. The atomic units, $h/2\pi = \hbar = c = 1$, are chosen. c is the velocity of light and h is the Planck constant. We use $\alpha = \sigma_3$ and $\beta = \sigma_1$, where σ_1 and σ_3 are Pauli matrices. Multiplying both sides of (1) by σ_1, one can explicitly write the

Dirac equation (1) in the form of

$$\left[-i\frac{d}{dx}\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}+V(x)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}+M(x)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\right]\Psi(x)=E\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\Psi(x). \tag{2}$$

The spinor wavefunction $\Psi(x)$ has two components. We denote the upper and lower components by $\phi(x)$ and $\theta(x)$, respectively. Equation (2) can be decomposed into the following two coupled differential equations

$$-i\frac{d\theta}{dx}+[E-V(x)]\theta-M\phi=0, \tag{3}$$

$$i\frac{d\phi}{dx}+[E-V(x)]\phi-M\theta=0. \tag{4}$$

Eliminating the lower spinor component form (3) and (4), we obtain a second order differential equation, which contains first order derivatives,

$$-\frac{d^2\phi}{dx^2}+V_{\mathit{eff}}(x)\phi=[E^2-M^2]\phi, \tag{5}$$

where an effective potential $V_{\mathit{eff}}(x)$ is defined as

$$V_{\mathit{eff}}(x)=-V^2-i\frac{dV}{dx}+2EV(x). \tag{6}$$

Now let us focus our attention on the PT-symmetric version of the kink-like potential in the form of

$$V(x)=i\alpha\beta\tanh\alpha x, \tag{7}$$

where α and the dimensionless coupling constant, β, are real numbers. This PT-symmetric potential is invariant under the change $\alpha\to-\alpha$ so that the results can depend only on $|\alpha|$.

Inserting (7) into (6) leads us to obtain the following PT-symmetric effective potential

$$V_{\mathit{eff}}(x)=-(\beta^2-\beta)\alpha^2\sec h^2\alpha x+2i\alpha\beta E\tanh\alpha x+\alpha^2\beta^2. \tag{8}$$

This potential possesses a form of the PT-symmetric Rosen-Morse-like potential [44, 45]. Inserting (8) into (5) leads us to obtain the following a Schrödinger-like equation

$$-\frac{d^2\phi(x)}{dx^2}+(-(\beta^2-\beta)\alpha^2\sec h^2\alpha x+2i\alpha\beta E\tanh\alpha x)\phi(x)=\tilde{E}\phi(x), \tag{9}$$

where the effective energy $\tilde{E}$ is defined as $\tilde{E}=E^2-M^2-\alpha^2\beta^2$. Here, the Dirac equation with the PT-symmetric version of the kink-like potential has been mapped into a Schrödinger-like equation with the PT-symmetric Rosen-Morse-like potential. Writing the ground-state upper component of the Dirac spinor $\phi_0(x)$ in the form of $\phi_0(x)=\exp(-\int W(x)dx)$ and substituting it into (9), we arrive at the following non-linear Riccati equation for $W(x)$,

$$W^2(x)-\frac{dW(x)}{dx}=-(\beta^2-\beta)\alpha^2\sec h^2\alpha x+2i\alpha\beta E\tanh\alpha x-\tilde{E}_0, \tag{10}$$

where $\tilde{E}_0$ is the effective ground-state energy, and $W(x)$ can be called a superpotential by using the basic concepts of the supersymmetric quantum mechanics [46]. Putting the superpotential $W(x)$ in the form of

$$W(x) = A + B\tanh\alpha x, \tag{11}$$

and using this expression, we obtain the unnormalized ground-state upper component $\phi_0(x)$,

$$\phi_0(x) = e^{-Ax}(\cosh\alpha x)^{-B/\alpha}. \tag{12}$$

In view of the bound state boundary conditions that $\phi_0(x)$ vanishes when $x \to \pm\infty$, we have the restriction conditions: $B/\alpha > 0$ and $|A| < B$. Inserting (11) into (10) and comparing equal powers of two sides in (10), we obtain a set of equations

$$A^2 + B^2 = -\tilde{E}_0, \tag{13a}$$

$$B^2 + \alpha B = \alpha^2\beta^2 - \alpha^2\beta, \tag{13b}$$

$$2AB = 2i\alpha\beta E. \tag{13c}$$

Solving this set of (13a–13c), we arrive at the following relations

$$A = i\frac{\alpha\beta E}{B} = i\frac{\beta E}{\beta - 1}, \tag{14a}$$

$$B = \alpha\beta - \alpha, \tag{14b}$$

$$\tilde{E}_0 = -(A^2 + B^2) = \left(\frac{\alpha\beta E}{B}\right)^2 - B^2. \tag{14c}$$

With the help of the superpotential $W(x)$ given in (11), one can construct the following two supersymmetric partner potentials

$$\begin{aligned} V_{eff_+}(x) &= W^2(x) + \frac{dW(x)}{dx} \\ &= -\left(\frac{\alpha\beta E}{B}\right)^2 + B^2 - (B^2 - \alpha B)\sec h^2\alpha x + 2i\alpha\beta E\tanh\alpha x, \end{aligned} \tag{15a}$$

$$\begin{aligned} V_{eff_-}(x) &= W^2(x) - \frac{dW(x)}{dx} \\ &= -\left(\frac{\alpha\beta E}{B}\right)^2 + B^2 - (B^2 + \alpha B)\sec h^2\alpha x + 2i\alpha\beta E\tanh\alpha x. \end{aligned} \tag{15b}$$

Setting $a_0 = B, a_1$ is a function of a_0, i.e., $a_1 = f(a_0) = a_0 - \alpha$, one can easily verify that these two partner potentials $V_{eff_+}(x)$ and $V_{eff_-}(x)$ satisfy the following relationship

$$V_{eff_+}(x, a_0) = V_{eff_-}(x, a_1) + R(a_1), \tag{16}$$

where the remainder $R(a_1)$ is independent of x,

$$R(a_1) = \left[-\left(\frac{\alpha\beta E}{a_0}\right)^2 + a_0^2\right] - \left[-\left(\frac{\alpha\beta E}{a_1}\right)^2 + a_1^2\right].$$

Equation (16) shows that the two partner potentials $V_{eff_+}(y)$ and $V_{eff_-}(y)$ have similar shapes and possess shape invariance in the senses of [47]. For the shape-invariant-like potential $V_{eff_-}(x)$, the exact energy spectrum can be calculated by using the following supersymmetric WKB quantization condition [48]

$$\int_{x_L}^{x_R} \sqrt{\tilde{E}_n^{(-)} - W^2(x)}dx = n\pi, \quad n = 0, 1, 2, \ldots, \tag{17}$$

where the two turning points x_L and x_R are given by $\tilde{E}_n^{(-)} - W^2(x) = 0$. Combining the ideas of the supersymmetric quantum mechanics and the standard WKB approximation method, Comtet et al. [48] proposed the lowest order supersymmetric WKB approach, which can give the exact energy spectra for the shape-invariant potentials. Substituting the superpotential $W(x)$ given in (11) into the supersymmetric WKB quantization condition (17), we have

$$\int_{x_L}^{x_R} \sqrt{\tilde{E}_n^{(-)} - (A + B\tanh\alpha x)^2}dx = n\pi. \tag{18}$$

By introducing a new variable $y = \tanh\alpha x$ and after algebraic simplification, (18) can be expressed as

$$\int_{y_L}^{y_R} \frac{B}{\alpha}\frac{1}{1-y^2}\sqrt{(y-y_L)(y_R-y)}dy = n\pi, \tag{19}$$

where the two turning points are given by

$$y_L = -\frac{A}{B} - \frac{\sqrt{\tilde{E}_n^{(-)}}}{B} \quad \text{and} \quad y_R = -\frac{A}{B} + \frac{\sqrt{\tilde{E}_n^{(-)}}}{B}. \tag{20}$$

For computing the integral in (19), we use the integral expression [49]

$$\int_a^b \frac{1}{1-y^2}\sqrt{(y-a)(b-y)}dy = \frac{\pi}{2}[2 - \sqrt{(1-a)(1-b)} - \sqrt{(1+a)(1+b)}], \tag{21}$$

where the limits a and b are real, and $-1 < a < b < 1$. Comparing (19) with (21), and solving (19) for $\tilde{E}_n^{(-)}$ gives

$$\tilde{E}_n^{(-)} = A^2 + B^2 - \frac{A^2B^2}{(B-n\alpha)^2} - (B-n\alpha)^2. \tag{22}$$

From (10) and (15b), we have the following relation

$$\tilde{V}_{eff}(x) = -(\beta^2-\beta)\alpha^2\sec h^2\alpha x + 2i\alpha\beta E\tanh\alpha x = V_{eff_-}(x) + \tilde{E}_0. \tag{23}$$

With the help of (14), (22) and (23), we obtain the solution for $\tilde{E}$ in (9),

$$\tilde{E} = \tilde{E}_0 + \tilde{E}_n^{(-)} = -\frac{A^2B^2}{(B-n\alpha)^2} - (B-n\alpha)^2 = \frac{\beta^2E^2}{((n+1)-\beta)^2} - \alpha^2((n+1)-\beta)^2. \tag{24}$$

Inserting the expression $\tilde{E} = E^2 - M^2 - \alpha^2\beta^2$ into (24), we find the following relativistic energy spectrum for the PT-symmetric kink-like potential (7) in the context of the Dirac

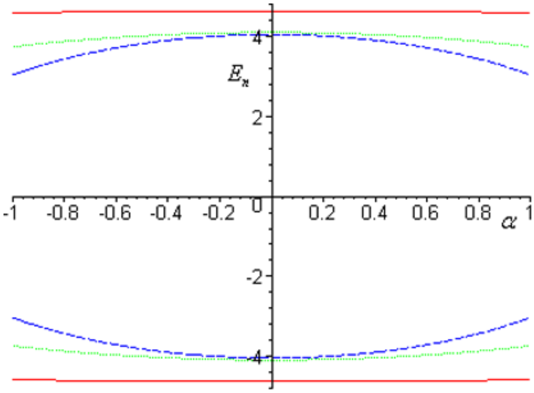

Fig. 1 (Color online) Dirac eigenvalues for the three lowest energy levels as a function of α for $\beta = 1/3$ and $M = 4$. The *solid red lines* stand for $n = 0$, the *dotted green lines* for $n = 1$ and the *dashed blue lines* for $n = 2$

equation with the vector potential coupling in $1 + 1$ dimensions,

$$E_n = \pm\left[\frac{\alpha^2((n+1)-\beta)^2 - \alpha^2\beta^2 - M^2}{\frac{\beta^2}{((n+1)-\beta)^2} - 1}\right]^{1/2}. \tag{25}$$

The real discrete spectrum consists of a positive series and a negative series. In order to hold a real energy spectrum, the part in the square bracket must be non-negative. This restriction condition leads us to demand that the quantum number n and three parameters must satisfy the conditions: $n = 0, 1, 2, \ldots < \beta - 1 + \frac{\sqrt{\alpha^2\beta^2+M^2}}{|\alpha|}$ and $\beta < 1/2$. The Dirac eigenenergies are plotted in Fig. 1 for the three lowest bound states as a function of α for $\beta = 1/3$ and $M = 4$. Figure 1 illustrates that the discrete positive energy spectra and negative energy spectra are symmetric about $E = 0$. Figure 1 also shows that the behavior of the Dirac eigenenergies depended only on $|\alpha|$.

Substituting (24) into (9), we obtain the following equation

$$-\frac{d^2\phi(x)}{dx^2} + (-(\beta^2 - \beta)\alpha^2 \sec h^2\alpha x + 2i\alpha\beta E \tanh\alpha x)\phi(x)$$
$$= \left(\frac{\beta^2 E_n^2}{((n+1)-\beta)^2} - \alpha^2((n+1)-\beta)^2\right)\phi(x). \tag{26}$$

Introducing the new variable $z = -\tanh\alpha x$ and writing the upper component $\phi(x)$ as $\phi(x) = (\frac{1-z}{2})^{-p}(\frac{1+z}{2})^{-w}P(z)$, (26) can be reduced to the following equation satisfied by $P(z)$,

$$(1-z^2)\frac{d^2P}{dz^2} + [-2w + 2p - (2 - 2p - 2w)z]\frac{dP}{dz} + n(n - 2p - 2w + 1)P = 0, \tag{27}$$

where p and w are defined as

$$p = \frac{1}{2}\left[n + 1 - \beta - \frac{i\beta E_n}{\alpha}\frac{1}{n+1-\beta}\right] \quad \text{and} \quad w = \frac{1}{2}\left[n + 1 - \beta + \frac{i\beta E_n}{\alpha}\frac{1}{n+1-\beta}\right],$$

respectively. Equation (27) is the well-known differential equation satisfied by the Jacobi polynomials $P_n^{-2p,-2w}(y)$, hence the upper spinor component $\phi(x)$ can be expressed as,

$$\phi_n(x) = \left(\frac{1+\tanh\alpha x}{2}\right)^{-p}\left(\frac{1-\tanh\alpha x}{2}\right)^{-w} P_n^{-2p,-2w}(-\tanh\alpha x). \tag{28}$$

After making some algebraic manipulations, we may rewrite the unnormalized upper spinor component $\phi_n(x)$ corresponding to energy level E_n in the fashion,

$$\phi_n(x) = (\cosh\alpha x)^{(p+w)} e^{\alpha(w-p)x} P_n^{-2p,-2w}(-\tanh\alpha x). \tag{29}$$

Substituting (29) into (3) and using the differential recursion relation for the Jacobi polynomials, we can obtain the lower spinor component corresponding to energy level E_n,

$$\begin{aligned}\theta_n(x) = \frac{1}{M}\bigg\{&\bigg[E_n + i\alpha(w-p) + i\frac{n\alpha(p-w)}{n-p-w} + i\alpha(-n-\beta+p+w)\tanh\alpha x\bigg]\phi_n(x)\\ &- i\frac{\alpha(n-2p)(n-2w)}{n-p-w}\phi_{n-1}(x)\bigg\}.\end{aligned} \tag{30}$$

When the relativistic energy spectra given in (25) are real, the upper spinor component $\phi_n(x)$ and lower spinor component $\theta_n(x)$ are simultaneously eigenstates of the PT operator: $\mathrm{PT}\phi_n(x) = \lambda_n\phi_n(x), \mathrm{PT}\theta_n(x) = \lambda_n\theta_n(x)$, where $\lambda_n = (-1)^n$. Because $(\mathrm{PT})^2 = 1$ and PT involves complex conjugation, it follows that $|\lambda_n| = 1$. If $\beta > 1/2$, first, the two sets of real energy spectra come into complex conjugate, second, the upper spinor component $\phi_n(x)$ and lower spinor component $\theta_n(x)$ are not simultaneously eigenstates of PT operator: $\mathrm{PT}\phi_n(x) \neq (-1)^n\phi_n(x)$ and $\mathrm{PT}\theta_n(x) \neq (-1)^n\theta_n(x)$. In this case, we say that PT symmetry is spontaneously broken. When $\beta > 1/2$, the effective energy $\tilde{E}$ is real, this is consistent with the fact that the effective potential $V_{eff}(x)$ given in (8) is real and Hermitian.

When PT symmetry is not spontaneously broken, the relativistic energy spectra E_n are real. In view of the relations: $p + w = n + 1 - \beta$ and $w - p = \frac{i\beta E_n}{\alpha}\frac{1}{n+1-\beta}$, we can find the conventional inner product for two upper spinor components is positive definite, namely that the integral

$$\langle\phi_n(x)|\phi_n(x)\rangle = \int_{-\infty}^{+\infty}\phi_n(x)[\phi_n(x)]^* dx, \tag{31}$$

is positive definite. However, the PT-inner product [50–52] of two upper spinor components is not positive definite, that is that the integral

$$\langle\phi_n(x)\mathrm{PT}|\phi_n(x)\rangle = \int_{-\infty}^{+\infty}\phi_n(x)[\phi_n(-x)]^* dx, \tag{32}$$

is not positive definite. This indefinite metric for the spinor wavefunctions also exists in the non-relativistic PT-symmetric quantum mechanics for the eigenvectors [50–52].

3 Conclusion

In conclusion, we may conclude that the Dirac equation for the bound states of the PT-symmetric kink-like potential in 1 + 1 dimensions can be solved exactly by using the basic concepts of the supersymmetric WKB formalism and function analysis method.

The Dirac equation with the PT-symmetric kink-like potential is mapped into the exactly solvable Schrödinger-like equation with the PT-symmetric Rosen-Morse-like potential. The PT-symmetric kink-like potential is not Hermitian and absent of bound states in the context of non-relativistic Schrödinger equation, however it possesses real discrete relativistic energy spectra in the context of the Dirac theory with the vector potential. When the PT symmetry is spontaneously broken, two sets of real energy spectra come into complex conjugate, and the upper spinor component $\phi_n(x)$ and lower spinor component $\theta_n(x)$ are also not the eigenfunctions of the PT operator. The PT-inner product of two upper spinor components is not positive definite.

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